Amplitude Modulation[1]

Both AM and SSB are examples of simple amplitude modulation. Consider the following from Wikipedia:

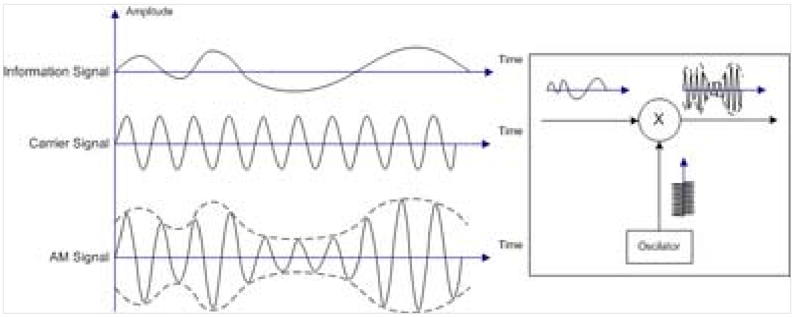

Illustration of Amplitude Modulation

Consider a carrier wave (sine wave) of frequency f_c and amplitude A given by:

$$c(t) = A \cdot \sin(2\pi f_c t).$$

Let $m(t)$ represent the modulation waveform. For this example we shall take the modulation to be simply a sine wave of a frequency f_m, a much lower frequency (such as an audio frequency) than f_c:

$$m(t) = M \cdot \cos(2\pi f_m t + \phi),$$

where M is the amplitude of the modulation. We shall insist that $M<1$ so that $(1+m(t))$ is always positive. If $M>1$ then over-modulation occurs and reconstruction of message signal from the transmitted signal would lead in loss of original signal. Amplitude modulation results when the carrier $c(t)$ is multiplied by the positive quantity $(1+m(t))$:

$$\begin{aligned} y(t) &= [1 + m(t)] \cdot c(t) \\ &= [1 + M \cdot \cos(2\pi f_m t + \phi)] \cdot A \cdot \sin(2\pi f_c t) \end{aligned}$$

In this simple case M is identical to the modulation index, discussed below. With M=0.5 the amplitude modulated signal $y(t)$ thus corresponds to the top graph (labelled "50% Modulation") in Figure 4.

Using prosthaphaeresis identities, $y(t)$ can be shown to be the sum of three sine waves:

$$y(t) = A \cdot \sin(2\pi f_c t) + \frac{AM}{2}\left[\sin(2\pi(f_c + f_m)t + \phi) + \sin(2\pi(f_c - f_m)t - \phi)\right].$$

Therefore, the modulated signal has three components: the carrier wave $c(t)$ which is unchanged, and two pure sine waves (known as sidebands) with frequencies slightly above and below the carrier frequency f_c.

The sum and difference comes from the trigonometric identity:

---

1 2015-2019 Element 3 Question Pool, revised 11 February 2015

$$\cos u \times \sin v = \frac{1}{2} \times [\sin(u+v) - \sin(u-v)]$$.

The take away is that carries frequency, f_c , and the two sidebands, $f_c + f_m$ and $f_c - f_m$ are produces in a 2 to 1 to 1 ratio.

It can be shown that AM produces a signal with ½ the power in the carrier component, ¼ of the power in each side component, ie upper-sideband and lower-sideband. Single-sideband optimizes the power lose by eliminating the carrier and one sideband and only power amplifying the remaining sideband. Note LSB uses the minus sign, $f_c - f_m$, since it lies below the carrier frequency and USB uses the plus sign, $f_c + f_m$, since it lies above the carrier frequency.

Questions

G4D08 (C)
What frequency range is occupied by a 3 kHz LSB signal when the displayed carrier frequency is set to 7.178 MHz?

A. 7.178 to 7.181 MHz
B. 7.178 to 7.184 MHz
C. 7.175 to 7.178 MHz
D. 7.1765 to 7.1795 MHz

The display is always the carrier frequency, f_c , even though it is not present in single-sideband. We shall also assume modulation frequency, $f_m = 3\,KHz = 0.003\,MHz$, since it is furthest from the carrier frequency. Since the lower-sideband is being used it goes from $f_C - f_{LSB} = 7.178 - 0.003 = 7.175\,\text{MHz}$ to $f_c = 7.178\,MHz$. Therefore C is the correct number.

G4D09 (B)
What frequency range is occupied by a 3 kHz USB signal with the displayed carrier frequency set to 14.347 MHz?

A. 14.347 to 14.647 MHz
B. 14.347 to 14.350 MHz
C. 14.344 to 14.347 MHz
D. 14.3455 to 14.3485 MHz

Again the display is always the carrier frequency even though it is not present in single-sideband. We shall also assume $f_m = 3\,KHz = 0.003\,MHz$, since it is furthest from the carrier frequency. Since the upper-sideband is being used it goes from $f_c = 14.347\,MHz$ to $f_C + f_{USB} = 14.347 + 0.003 = 14.350\,\text{MHz}$. Therefore B is the correct number.

G4D10 (A)
How close to the lower edge of the 40-meter General Class phone segment should your displayed carrier frequency be when using 3 kHz wide LSB?

A. At least 3 kHz above the edge of the segment

B. At least 3 kHz below the edge of the segment
C. Your displayed carrier frequency may be set at the edge of the segment
D. At least 1 kHz above the edge of the segment

Assume that the far reach of the 3 kHz LSB is at the lower edge the 40-meter General Class phone segment, then $f_{\text{lower edge}} = f_c - 3\,\text{kHz}$. Now solve for the carrier frequency, thus $f_c = f_{\text{lower edge}} + 3\,\text{kHz}$. The minimum requirement is therefore 3 Khz above the lower edge. The correct answer is A.

G4D11 (B)
How close to the upper edge of the 20-meter General Class band should your displayed carrier frequency be when using 3 kHz wide USB?
A. At least 3 kHz above the edge of the band
B. At least 3 kHz below the edge of the band
C. Your displayed carrier frequency may be set at the edge of the band
D. At least 1 kHz below the edge of the segment

Assume that the far reach of the 3 kHz USB is at the upper edge the 20-meter General Class phone segment, then $f_{\text{upper edge}} = f_c + 3\,\text{kHz}$. Now solve for the carrier frequency, thus $f_c = f_{\text{upper edge}} - 3\,\text{kHz}$. The minimum requirement is therefore 3 Khz below the upper edge. The correct answer is B.

Frequency Modulation

The mathematical rule for frequency modulation is quite complex requiring the use of Bessel functions. That is why it is not presented here. But the bottom line for hams is Carson's rule. Carson's rule states that 98% power in an FM signal is within the bandwidth, B_T, where $B_T = 2 \times (f_{\text{deviation}} + f_m)$.

Frequency modulated radios in the VHF bands and above often use the harmonic of the base frequency oscillator. The n-th harmonic is the base frequency multiplied by n, $f_n = n \times f_{\text{base}} = n\, f_{\text{base}}$. Therefore for the first harmonic n = 1, $f_1 = 1 \times f_{\text{base}} = f_{\text{base}}$, or the base frequency itself. The second harmonic is, $f_2 = 2 \times f_{\text{base}} = 2\, f_{\text{base}}$, the third harmonic is $f_3 = 3 \times f_{\text{base}} = 3\, f_{\text{base}}$, etc. To get the harmonic of interest, some wave shaping is performed, then the signal is filtered for the desired frequency and then amplified. The desired wave shaping is selected using Fourier analysis which is beyond the scope of this solution set.

Questions

G8B06 (D)
What is the total bandwidth of an FM phone transmission having 5 kHz deviation and 3 kHz modulating frequency?
A. 3 kHz
B. 5 kHz
C. 8 kHz
D. 16 kHz

From Cramer's rule we have $B_T = 2\times(5+3) = 2\,x\,8 = 16\,\text{kHz}$. Therefore the answer is D. You either have to do some rather complex math on the exam or remember that the bandwidth is twice the deviation frequency plus the modulation frequency.

```
G8B07 (B)
What is the frequency deviation for a 12.21 MHz reactance modulated
oscillator in a 5 kHz deviation, 146.52 MHz FM phone transmitter?
   A. 101.75 Hz
   B. 416.7 Hz
   C. 5 kHz
   D. 60 kHz
```

This problem is a two step problem. In this problem we must first determine what harmonic of the 12.21 Mhz fundmental harmonic is used in the nth harmonic for 146.52 Mhz. That is $f_n = n\times(12.21) = 146.52$. Now we solve for n, $n = 146.52/12.21 = 12^{th}$ harmonic. In the second step the 5 kHz deviation at 146.52 Mhz must be 12^{th} of that at 12.21 Mhz. Therefore $f_{\text{deviation @ 12.21}} = \frac{f_{\text{deviation @ 146.52}}}{12} = 5/12 = 0.4167\,\text{KHz} = 416.7\,\text{Hz}$. Therefore B is the correct answer. Everything scales up by the factor of 12, the carrier frequency and the deviation when the 12^{th} harmonic of the reactance modulator is used.